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Computer Analysis of Feynman Diagrams for a Static Model*

A computer program, written in FORTRAN, which constructs and computes Feynman diagrams for the charged scalar static model is described.

Computations in field theory are complicated by the large number and complexity of Feynman diagrams necessary to describe the higher orders of perturbation theory. Campbell and Hearn have described a very sophisticated system of computer programs designed to handle this problem for any interaction [1]. The purpose of this letter is to describe a program which performs the more limited task of constructing and computing the bare perturbation series diagrams for the charged-scalar meson field interacting with a static nucleon source [2]. This is not a trivial problem, since, for example, even the two-point (propagator) function has 71 graphs in fifth order and 461 in sixth order. Power series expansions computed with this program may be used, for example, to study the application of Padé approximants to the charged scalar static model [3]. The program is written in FORTRAN and may be conveniently subdivided into two sections, one which constructs the diagrams and sets up the integrals for the matrix elements and one which numerically evaluates these integrals.

The details of the noncovariant perturbation theory for this static model are presented in Ref. [4]. The case of π^+p scattering is considered here, although the program is easily generalized to other situations. For a given order in $\lambda = g_0^2$, the quantities which characterize a given diagram in the perturbation theory are the propagation denominators arising in each *slot* between vertices.

In particular, the contribution of a given diagram depends upon which mesons are present in each slot. For example, the graph in Fig. 1(a) is characterized by a denominator composed of the factors

$$D_a = (V - \omega_A - \omega_1)(V - \omega_1)(V - \omega_1 - \omega_2)(V - \omega_2)(V - \omega_B - \omega_2),$$

where $V = E - m_0$ (m_0 is the bare mass) and the various ω's are the energies for each meson $\omega = \sqrt{k^2 + \mu^2}$. This graph is the only graph in which the ω's appear

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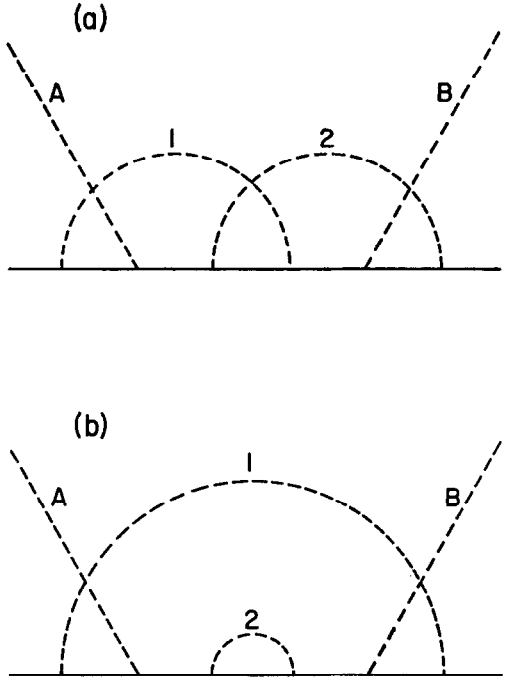

FIG. 1. Two third-order (in $\lambda = g_0^3$) Feynman graphs for the π^+p scattering amplitude in the charged scalar static model are shown. These graphs are two different branches of the same $N = 3$ class and sequence.

in this particular order. The set of all allowed combinations of the ω's is in one-to-one correspondence with the graphs in that order of perturbation theory.

Given N, the desired order of perturbation theory, construction of the Feynman diagrams and the corresponding integrals is accomplished by three levels of partitioning of diagrams according to various structural features of the graphs. The first partition sorts diagrams according to the particular attachment of the external meson lines in the graph. Two of the $2N$ vertices in the N-th order (in λ) correspond to emission and adsorption of external mesons. There are, therefore, N^2 possible arrangements of these external lines; each such arrangement is called a *class*. A class is numerically represented by an array of $2N - 1$ integers denoting the number of external mesons present in each slot. For the graphs in Fig. 1, for example, the class would be represented by the vector $(1, 0, 0, 0, 1)$. This array serves as input to the subsequent sections of the program and is also used to correctly insert external meson energies in the final calculation.

Each class is next partitioned into *sequences*. A sequence is characterized by two arrays of $2N$-1 integers corresponding to the number of internal (virtual) mesons

of a particular charge present in each slot. For example, the sequence for the positive mesons for both graphs in Fig. 1 would be (1, 1, 2, 1, 1), while that for the negative mesons would be (0, 0, 0, 0, 0). The sequences are constructed by allowing all possible emissions and absorptions of internal mesons subject to the following constraints:

(a) charge is conserved; (b) no internal mesons are present prior to the first or subsequent to the final vertex interaction; (c) improper graphs (having a bare internal nucleon) are forbidden; (d) external emissions and absorptions take precedence over internal meson events.

The two arrays of integers computed here are the input for the next section.

The final partititon involves diagrams having the same class and sequence structure. Whenever a meson is absorbed and more than one appropriately charged meson is present in the preceding slot, an independent diagram is created for each possible absorption. Both graphs in Fig. 1 have the same class-sequence structure but differ in which meson is absorbed between the third and fourth slots. Since each emitted meson represents a unique integration variable, care must be taken to distinguish between mesons used at various places in the diagram. Logical arrays are used to store the past histories of the virtual mesons and to determine what mesons are available for absorption at any given vertex. These logical arrays are then used to create two arrays of integers (one for each charge), which are used in the actual diagram calculation; these integers are equal to 1 if a particular meson is present in a particular slot of the *branch* or diagram being considered and are 0 otherwise. If we denote the array of integers for positive mesons by Q (meson number, slot), the values of Q for the diagram in Fig. 1(a) are given by: $Q(1, 1) = 1$, $Q(1, 2) = 1$, $Q(1, 3) = 1$, $Q(1, 4) = 0$, $Q(1, 5) = 0$, $Q(2, 1) = 0$, $Q(2, 2) = 0$, $Q(2, 3) = 1$, $Q(2, 4) = 1$, $Q(2, 5) = 1$. This array plus the class vector given above are used to set up the denominator D_α. The contribution of this diagram is then given by

$$\lambda^3 \int_1^\infty \int_1^\infty \frac{\sigma(\omega_1)\, \sigma(\omega_2)\, d\omega_1\, d\omega_2}{D_\alpha(\omega_1, \omega_2)},$$

where σ is the phase space-cutoff function.

When the integration dimensionality ($N - 1$) becomes greater than 4, it is more economical to use Monte Carlo method than simple quadratures for evaluation of the noncovariant Feynman integrals. The integration region $[1, \infty]$ is mapped onto $[0, 1]$ by a linear fractional transformation with one free parameter. This parameter is adjusted to give maximum accuracy. Several methods of importance sampling were attempted but did not improve the results. Experience indicates that 5000 sets of random values for the transformed internal meson energies are sufficient to obtain results accurate to within 1 %.

While this particular program is limited compared to the system in Ref. [1], the direct analysis of the graph structure used here could be useful when applied to other field theories. Additional information and listings are available from the authors.

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